Advanced Division Accuracy Round Solutions

1. What is the unit digit of $3^7 + 7^3$?

(a) 0 (b) 2 (c) 4
(d) 6 (e) 8

Solution: $3^7 = 2187$ and $7^3 = 343$. $2187 + 343 = 2530$, so the units digit is 0. **Answer: (A) 0**

2. There are many options for girls' sports. In the fall, a school offers Cross-Country, Field Hockey, Pom Pon, Sideline Cheer, Tennis, and Volleyball. In the winter, there is Competitive Cheer, Basketball, Wrestling, and Swim & Dive. In the spring, Lacrosse, Soccer, Softball, and Track are presented. One can also choose not to participate in any sport in a particular season. How many ways can a student choose her sports for the year, assuming she can participate in no more than one sport per season?

(a) 45 (b) 96 (c) 175
(d) 225 (e) 256

Solution: In fall, there are 6 sports and the option to not do a sport, meaning 7 choices. We see that winter has 4 sports and the option to not do a sport, so 5 choices. Spring, similar to winter, will also have 5 choices. There are $7 \cdot 5 \cdot 5 = 175$ ways. **Answer: (C) 175**

3. Boys' athletics just added a new sport: Volleyball! If there are 6 players on a starting court who will eventually rotate positions, how many ways can they arrange themselves if rotations of arrangements are counted as the same?

(a) 120 (b) 96 (c) 720
(d) 24 (e) 200

Solutions: Since the court will rotate, we can treat this arrangement like a circle. The number of ways to organize the 6 people around the circle is $(6-1)! = 5! = 120$. **Answer: (A) 120**

4. The speed of a vehicle is increasing at a rate of 70 $\frac{m}{s^2}$. Convert this rate to $\frac{km}{min^2}$.

(a) 67 (b) 140 (c) 400
(d) 360 (e) 252

Solution: This is a classic unit conversion problem:

$$70\frac{m}{s^2}\cdot\frac{1}{1000}\frac{km}{m}\cdot(60^2)\frac{s^2}{min^2}=252\frac{km}{min^2}$$

Answer: (E) 252

5. The speed of light is, to the nearest integer estimate, 299,792,458 meters per second. Two prime numbers under ten can divide that number. What is the sum of those two factors?

(a) 5 (b) 7 (c) 8
(d) 9 (e) 12

Solution: For this problem, it is helpful to know the divisibility rules (search these up!) of the prime numbers less than 10. We see that 299,792,458 is divisible by 2 and 7, so the answer is $2+7=9$. To clarify, the divisibility rule for 2 is that the last digit must be even; the rule for 7 is that double the last digit, minus the remaining number, must be a multiple of 7. You can watch a YouTube video for further clarification. **Answer: (D) 9**

6. What is the area of a 13,14,15 triangle?

(a) 72 (b) 84 (c) 96
(d) $6\sqrt{105}$ (e) $84\sqrt{3}$

Solution: The most straightforward way is to use Heron's formula: $area=\sqrt{s(s-a)(s-b)(s-c)}$, where s is the semiperimeter (half of the triangle's perimeter) and $a,b,$ and c represent the lengths of the sides. The semiperimeter is $\frac{13+14+15}{2}=21$. The area will be $\sqrt{(21)(6)(7)(8)}=84$. **Answer: (B) 84**

7. What is the probability you roll a die three times such that the product of the first two values equals the third value?

(a) $\frac{7}{108}$ (b) $\frac{2}{27}$ (c) $\frac{1}{12}$
(d) $\frac{5}{54}$ (e) $\frac{1}{18}$

Solution: We can start by simply listing the triples that satisfy this condition: (1,1,1), (1,2,2), (1,3,3), (1,4,4), (1,5,5), (1,6,6), (2,1,2), (2,2,4), (2,3,6), (3,1,3), (3,2,6), (4,1,4), (5,1,5), (6,1,6), for a total of 14 such triples. There are $6^3=216$ triples that can be obtained by rolling the die 3 times, giving us $\frac{14}{216}=\frac{7}{108}$. **Answer: (A)** $\frac{7}{108}$

8. The number 2025 CANNOT be expressed in which of the following ways:

(a) $(\sum_{n=1}^{9} n)^2$ (b) $\sum_{n=1}^{9} n^3$ (c) $(20+25)^2$
(d) 20^3+25^3 (e) $(\sum_{n=1}^{9} 5)^2$

Solution: To start, we need to know what the $\sum$ (sum) symbol means. The $\sum$ denotes the sum of all terms n starting with the first integer (on the bottom of the sum symbol) until the last number (on the top of the sum symbol). By this logic, we see that option A is $(1+2+...+9)^2 = 2025$, option B is $1^2+2^3+...+9^3 = 2025$, option C is $45^2 = 2025$, option D is $20^3+25^3 = 23625$, and option E is $(9\cdot 5)^2 = 45^2 = 2025$. **Answer: (D)** 20^3+25^3

9. If r and s are the roots of the quadratic x^2+6x+4. What is the difference between r and s?

(a) 6 (b) 4 (c) $2\sqrt{5}$
(d) 12 (e) $2\sqrt{2}$

Solution: The roots of a quadratic are given by $\frac{-b\pm\sqrt{b^2-4ac}}{2a}$, and the difference of the roots will be $\frac{\sqrt{b^2-4ac}}{a} = \frac{\sqrt{6^2-4(1)(4)}}{1} = \frac{\sqrt{20}}{1} = 2\sqrt{5}$. **Answer: (C)** $2\sqrt{5}$

10. What is the ratio of the circumradius of an equilateral triangle to the inradius of the same equilateral triangle?

(a) 1 (b) $\sqrt{3}$ (c) 2
(d) $\frac{2}{\sqrt{3}}$ (e) 3

Solution: For an equilateral triangle with side a, the circumradius is $R = \frac{a}{\sqrt{3}}$ and the inradius is $r = \frac{a}{2\sqrt{3}}$. Therefore, the ratio is $\frac{R}{r} = \frac{\frac{a}{\sqrt{3}}}{\frac{a}{2\sqrt{3}}} = 2$. **Answer: (C) 2**

11. A regular hexagon has a circumcenter O. The distance from O to any of the sides is $3\sqrt{3}$. What is the area of the hexagon?

(a) 27 (b) $27\sqrt{3}$ (c) 54
(d) $54\sqrt{3}$ (e) $81\sqrt{3}$

Solution: In a regular hexagon, the distance from the center to a side equals the inradius, given by $r = \frac{\sqrt{3}}{2}a$. Thus, $3\sqrt{3} = \frac{\sqrt{3}}{2}a \Rightarrow a = 6$. The area of a regular hexagon is $\frac{3\sqrt{3}}{2}a^2 = \frac{3\sqrt{3}}{2}(6)^2 = 54\sqrt{3}$.

Answer: (D) $54\sqrt{3}$

12. How many ways can I arrange the digits of 12345 so the even digits are in increasing order?

(a) 20 (b) 60 (c) 70
(d) 120 (e) 80

Solution: The even digits 2 and 4 must remain in increasing order. We choose any 2 of the 5 positions to place them, which can be done in $\binom{5}{2} = 10$ ways. The remaining 3 digits (1, 3, 5) can be arranged freely in $3! = 6$ ways. Therefore, the total number of arrangements is $10 \times 6 = 60$. **Answer: (B) 60**

13. It takes Benny 4 hours to paint a wall. Jenny can finish the wall in 6 hours. If Benny starts painting at 8 am one day and Jenny joins him at 10 am, what time will they finish?

(a) 10:50 am (b) 11:12 am (c) 12 noon
(d) 12:11 pm (e) 11:30 am

Solution: Benny paints at a rate of $\frac{1}{4}$ of the wall per hour, while Jenny paints at $\frac{1}{6}$ per hour. By 10 am, Benny has worked 2 hours and completed $\frac{1}{2}$ of the wall. Together, their combined rate is $\frac{1}{4} + \frac{1}{6} = \frac{5}{12}$ per hour. The remaining $\frac{1}{2}$ of the wall takes $\frac{1/2}{5/12} = \frac{6}{5} = 1.2$ hours, or 1 hour and 12 minutes. Thus, they finish at 11:12 am.

Answer: (B) 11:12 am

14. A code is formed by arranging 3 different letters chosen from A, B, C, D, E. How many such codes contain at least one vowel (A or E)?

(a) 60 (b) 12 (c) 36
(d) 48 (e) 54

Solution: The total number of 3-letter codes using A–E is $5P3 = 60$ (aPb $= \frac{a!}{(a-b)!}$). To find those with at least one vowel (A or E), subtract codes with no vowels. Using only B, C, and D gives $3P3 = 6$. Hence, $60 - 6 = 54$ codes have at least one vowel. **Answer: (E) 54**

15. If you roll 3 fair, six-sided dice, what is the probability of obtaining a sum of 8?

(a) 7/72 (b) 7/60 (c) 6/7
(d) 3/8 (e) 3/5

Solution: The number of ordered triples (a, b, c) of die rolls with sum 8 is 21 (a standard count). There are $6^3 = 216$ total outcomes. We need to count the number of ordered triples (a, b, c) such that

$$a + b + c = 8, \qquad a, b, c \in \{1, 2, 3, 4, 5, 6\}.$$

Fix a and determine how many pairs (b, c) satisfy $b + c = 8 - a$.

a	Possible (b, c) pairs
1	$(1,6),(2,5),(3,4),(4,3),(5,2),(6,1) \Rightarrow 6$
2	$(1,5),(2,4),(3,3),(4,2),(5,1) \Rightarrow 5$
3	$(1,4),(2,3),(3,2),(4,1) \Rightarrow 4$
4	$(1,3),(2,2),(3,1) \Rightarrow 3$
5	$(1,2),(2,1) \Rightarrow 2$
6	$(1,1) \Rightarrow 1$

Adding these gives the total number of favorable outcomes:

$$6 + 5 + 4 + 3 + 2 + 1 = 21.$$

The probability is $\frac{21}{216} = \frac{7}{72}$. **Answer: (A)** $\dfrac{7}{72}$

16. Given the system of equations $x + 2y = 5$, $2x + 4y = 10$, for any given value of a, which of the following points lies on the graph of both equations?

(a) (a) (a, $\frac{5}{2} + a$) (b) (4-a, a+$\frac{1}{2}$) (c) (a+$\frac{1}{2}$, a)
(d) (5,10) (e) (a, (5-a)/2)

Solution: The second equation $2x + 4y = 10$ is 2 times the first equation $x + 2y = 5$, so they represent the same line. Every point on the line satisfies both equations. Solving for y in terms of x gives $y = (5 - x)/2$. Thus a general point on the common graph is $\big(a,\, (5 - a)/2\big)$.

Answer: (E) $\big(a,\, (5 - a)/2\big)$

17. Freddy the frog has four lily pads as his home. He can jump from any one to any other one, but every jump must be a jump to another lily pad, meaning he cannot stay at the current lily pad. After making 3 jumps, how many ways can he jump if he is back on his original lily pad before he jumped these 3 jumps?

(a) 4 (b) 6 (c) 8
(d) 12 (e) 16

Solution: Label the starting pad V. On the first jump Freddy must go to one of the other 3 pads. On the second jump he must go to a pad that is not V (otherwise the third jump cannot return to V since staying is not allowed), so for a chosen first pad there are 2 choices for the second pad. Finally the third jump is forced back to V. Thus there are $3 \cdot 2 \cdot 1 = 6$ possible sequences.**Answer: (B) 6**

18. Pascal's triangle is an arrangement in which each number in the rows below the first is formed as the sum of the 2 numbers above it. The 0th row has one number which is 1, the 1st row has 2 numbers which are both 1, the 2nd row has 3 numbers, namely 1, 2, and 1, so on so forth. In which row does the number 3003 appear first?

(a) 8 (b) 14 (c) 91
(d) 77 (e) 3003

Solution: We seek the first (0-indexed) row of Pascal's triangle containing 3003. For this problem, the easiest approach is just to draw out Pascal's Triangle and solve until you reach the fourteenth row. Another way to do this problem is just to use binomial theorem, with a brief explanation below: Each term of the expansion has the form $\binom{n}{k}a^{n-k}b^k$, where $\binom{n}{k} = \frac{n!}{k!(n-k)!}$ is the **binomial coefficient** (read as "n choose k").

The exponents follow a simple pattern:

- The power of a starts at n and decreases to 0.
- The power of b starts at 0 and increases to n.

Expanding $(a+b)^{14}$ would give us 3003 for the first time. Please note that this problem was INTENDED to be difficult to solve. **Answer: (B) 14**

19. Points P (3,4), Q (6,7), and R (m,n) are on the same line. Given QR = 3PQ and that (m,n) lies in Quadrant III, what is the value of m+n?

(a) -13 (b) -11 (c) -5
(d) 6 (e) -17

Solution: We use vector geometry: $\overrightarrow{PQ} = (6-3,\, 7-4) = (3,3)$. If QR = 3PQ and R is in Quadrant III, we reasonably conclude that P is between Q and R, so RP = 2PQ. Doing some arithmetic, we see that $m = 3 - 2(6-3) = -3$ and $n = 4 - 2(7-4) = -2$, so $m + n = -5$
Answer: (C) -5

20. In a certain tournament, there are 2 groups of 4 teams each. During the group stage, each team will play every other team in its group once. There will be a Super Four round with the top 2 teams from each group, where each of the 4 qualified teams will play against each other once. There will be a final match between the top 2 teams in the Super Four. How many matches will there be in total, assuming no matches get canceled?

(a) 55 (b) 23 (c) 19
(d) 48 (e) 25

Solution: Each group of 4 teams has $\binom{4}{2} = 6$ matches, so the group stage has $2 \cdot 6 = 12$ matches. The Super Four (4 teams) has $\binom{4}{2} = 6$ matches. The final adds 1 match. In total $12 + 6 + 1 = 19$ matches.

Answer: (C) 19